

Exploring variations in the gauge sector of a six-dimensional flavour model

Jean-Marie Frère^a, Maxim Libanov^{b,c}, Simon Mollet^a and Sergey Troitsky^b

^a*Service de Physique Théorique,*

Université Libre de Bruxelles, ULB, 1050 Brussels, Belgium

^b*Institute for Nuclear Research of the Russian Academy of Sciences,*

60th October Anniversary Prospect 7a, 117312, Moscow, Russia

^c*Moscow Institute of Physics and Technology,*

Institutskii per., 9, 141700, Dolgoprudny, Moscow Region, Russia

Abstract

We address the question, how general is the gauge sector in extra-dimensional models which explain hierarchies of masses and mixings of quarks, charged leptons and neutrinos in terms of a single family of multidimensional fermions. We give qualitative arguments that though there are a plethora of possible variations, they do not result in drastical changes of phenomenology.

1 Introduction

The wonderful world of large and infinite extra dimensions (ED), where low-energy excitations of multidimensional fields (“zero modes”) are bound to a (3+1)-dimensional manifold (“the brane”) representing our world, was discovered for theoretical physicists in independent works of Rubakov and Shaposhnikov [1], Akama [2] and Visser [3] more than four decades ago. Since then, enlarged symmetries of multidimensional worlds have been exploited in field-theory frameworks to address various fine-tuning and hierarchy problems of the Standard Model (SM) of particle physics, see e.g. reviews [4, 5] and

references therein. One of the approaches transfers geometric symmetries of the extra dimensions into flavour symmetries of our world, explaining in an elegant way the hierarchy of masses and mixings of SM quarks and charged leptons [6, 7, 8] and leading to rich testable phenomenology [9, 10, 11, 12]. The same model explains as well a very different pattern of neutrino masses and mixing, the difference with quarks being caused by the Majorana form of the neutrino mass term [13] (see Ref. [14] for a recent update). The purpose of the present work is to explore some ways beyond the simplest model and to sketch how robust its predictions are.

In ED models that hope to embed the SM, some vector fields must be introduced which will play the role of usual gauge fields at low energy. Their (almost) massless “zero” modes appear as the usual (3+1)-dimensional (4D) gauge bosons. The way of implementing a mechanism responsible for that is not always an easy task for there are further requirements to build a realistic model. Indeed, while we want the gauge zero mode to interact properly with the fermionic ones, we know that there will also exist a set of heavier (excited) modes which should not talk *too much* with this low energy sector, *i.e.* either there must exist a mass gap or these modes must only interact very weakly with the low-energy sector [15]. On the other hand, these new modes could manifest themselves at higher energy (in collider experiments for instance) or in (very) rare processes (e.g., flavour-changing neutral currents), thus providing hints for this kind of models.

In this note, we would like to provide with a short update of the constraints from these experiments for various models of this kind. We will focus on a particular class in (4+2) dimensions where a Nielsen-Olesen vortex-like defect plays the role of our 4D world [6, 7, 8, 15, 16]. We know that, quite generally in this background, we can get several localized (chiral) fermionic zero modes from a single spinor in 6D [17], each of them associated with a different winding in ED¹ ($e^{iw\varphi}, e^{i(w+1)\varphi}, e^{i(w+2)\varphi}, \dots$). They can acquire (small) masses through the vacuum expectation value of a Brout-Englert-Higgs (BEH) field H . In a certain range of parameters [12], the particular shape of this vev in ED (non zero in the core, almost zero outside) leads to a hierarchical pattern of masses. This idea was exploited in different contexts to reproduce the three SM generations and their spectrum. Here however we will only be interested in their interactions with gauge

¹The exact values of the windings are not important. What will be really relevant for us are the difference in windings between two modes.

bosons (both zero and heavy modes).

In section 2, we come back on some possible ways of introducing gauge bosons in the model and try to convince the reader that the expected phenomenology should not change drastically from one realisation to the other. In particular we will recall the existence of heavy localized modes whose mass scale is set by the geometry. Unlike the zero mode, the former possess non zero windings and can therefore be responsible for flavour changing processes (even in the absence of mixing in the fermionic sector) [10, 11]. In section 3, we comment on these processes and provide with some numerical results for the precise realization of [14]. Finally we conclude in section 4.

2 Some generic examples

Let us here quickly remind some general results. We will focus on models with 4D Poincaré invariance and 4D flat space. The most general metrics of such kind can be written as [18]:

$$ds^2 = G_{AB}dx^A dx^B = \sigma(y)\eta_{\mu\nu}dx^\mu dx^\nu - \gamma_{ab}(y)dy^a dy^b \quad (1)$$

With the following choice of gauge:

$$\begin{cases} \partial_0 W_0 - \partial_i W_i = 0, \\ \frac{\partial_a \left(\sqrt{|G|} \sigma^{-1} \gamma^{ab} W_b \right)}{\sqrt{|G|} \sigma^{-1}} = 0, \end{cases}$$

we have the obvious separation of variables in the equation of motion (EOM) for vector modes:

$$W_\mu(x, y) = \sum_n \omega_{\mu,n}(x) P_n(y)$$

with the modal wavefunctions P_n satisfying

$$\frac{\partial_a \left(\sqrt{|G|} \sigma^{-1} \gamma^{ab} \partial_b P \right)}{\sqrt{|G|} \sigma^{-1}} + \sigma^{-1} m^2 P = 0.$$

There always exists a zero mode ($m^2 = 0$) with a constant transverse wavefunction ($P(y) = \text{const}$), but we cannot conclude, at this level, if it is normalizable or not.

Two ways to ensure the normalizability are (i) to deal with compact ED whose finite volume renders the integral with the constant delocalized wavefunction bounded, or (ii) to make use of warp factors [19, 20, 21, 22] which will sufficiently "dilute" the wavefunction,

yet yield to a finite integral[23, 24]. Note that in the latter case, we can also consider effective wavefunctions in flat space which include warp factors and are thus localized from this point of view [18]. We will provide realizations of these two scenarios in the further simplified metrics, which is a particular case of (1):

$$ds^2 = \sigma(u)\eta_{\mu\nu}dx^\mu dx^\nu - du^2 - \gamma(u)dv^2.$$

A simple example of the first way (compact space) is the 2-sphere [8, 10, 11] of radius R which corresponds to $u = R\theta$, $v = R\varphi$ and $\gamma = \sin^2\theta$. The modal equation becomes then the equation for spherical harmonics with $R^2m^2 = \ell(\ell+1)$. As expected we have a (normalizable) zero mode $\ell = 0$ with constant wavefunction $P = 1/\sqrt{4\pi}R$. Heavier modes appear to be normalizable, too. The mass scale is dictated by the size of ED. In particular, there is a mass gap of the order $1/R$. For each value of ℓ , there are degenerate modes with windings $-\ell \leq m \leq \ell$. The wavefunctions oscillate on a scale of order R for the lightest modes.

If we opt instead for the warped case, the warp metrics can be parametrized [15] as $u = r$, $v = a\theta$, $\sigma = e^{A(r)}$ and $\gamma = e^{B(r)}$. The precise behaviour of the A and B functions are determined by the exact realization of the defect, but we can establish general features of their asymptotics by requiring (i) the metrics to be a regular solution of the 6D Einstein equations where a negative bulk cosmological constant balances a positive string tension (in the core)² and (ii) the gravity to be localized³. What we get is [15, 16] $A'(0) = 0$ and $B(r \rightarrow 0) \sim 2\ln(r/a)$ around the origin and $A = B = -2rc$ outside the core (c is a dimensional constant related to the bulk cosmological constant) which correspond to an AdS_6 geometry. We still have the arbitrariness of normalization and choose $A(0) = 0$. The dimensionfull constant, which will play an important role later on, a is not a free parameter but is determined by an interplay between the gravity and the vortex scales. With these asymptotics it is easy to realize that the two ED are a warped plane in polar coordinates and it is then obvious to further develop the P wavefunctions on a Fourier basis:

$$P_n(r, \theta) = \sum_{\ell} \rho_{n\ell}(r) e^{i\ell\theta}.$$

²Note that at 4D level we ask for a zero cosmological constant to have a flat space.

³*i.e.* ask for a normalizable zero mode for the graviton [25].

With this, the equation for ρ becomes:

$$\rho'' + \left(A' + \frac{B'}{2}\right)\rho' + \left(m^2 e^{-A} - \frac{\ell^2}{a^2} e^{-B}\right)\rho = 0.$$

Outside the core, the solutions are classified in terms of $\mu^2 = m^2 - \ell^2/a^2$. For $\mu = 0$, we have a constant solution, while for $\mu \neq 0$, it reads

$$\rho(r) = e^{\frac{3}{2}cr} \left[C_1 J_{3/2} \left(\frac{\mu}{c} e^{cr} \right) + C_2 Y_{3/2} \left(\frac{\mu}{c} e^{cr} \right) \right],$$

where J and Y are Bessel functions, and C 's are arbitrary constants. The boundary conditions (absence of the flux at infinity) lead to a continuous spectrum for $\mu > 0$ [26]. If we use the expression of J and Y in terms of elementary functions, it is easy to show that ρ behaves as ηe^{cr} at sufficiently large r , where η is some oscillating and bounded function. Now remember that, in the initial action, we have a factor $\sim \sqrt{|g|}(g^{00})^2 = a e^{B/2} \sim e^{-cr}$ for the kinetic term of 4D gauge component (and the integral over r fixes the normalization). As announced, we can define an effective wavefunction that takes this warp factor into account, then we can conclude if the associated mode is localized or not. With the definition $\zeta(r) = e^{-\frac{c}{2}r} \rho(r)$, we see that for the "constant" mode $\zeta_0(r) \sim e^{-\frac{c}{2}r}$ is localized⁴, while the continuous spectrum $\zeta_c(r) \sim \eta e^{\frac{c}{2}r}$ is not. The "not localized" states have most of their weight at large distances (therefore reducing the overlap). Now near the origin, the regular solution is:

$$\rho(r) \sim J_\ell(mr),$$

For $m = \ell/a$ (corresponding to localized mode $\mu = 0$ at infinity) we have (note that here, the metric factor is simply r):

$$\rho_0(r) \sim J_\ell \left(\ell \frac{r}{a} \right).$$

For $\ell = 0$ we get the usual constant solution (which matches with the constant solution at infinity, since we know that $\rho = \text{const}$ is an exact solution for the all range of r). For non-zero ℓ , we cannot get an exact solution, but we see that (at least for the first modes) we have oscillating functions with a scale of oscillation of order a .

In conclusion, we have a pattern which looks very much like the spherical case: discrete (localized) modes with mass scale $1/a$ and this same scale giving also an idea of the

⁴Note that in the usual 5D Randall-Sundrum models, this zero mode is not normalizable because the e^B factor is not present. The presence of an extra warped dimension helps to "dilute" more efficiently the constant wavefunction.

oscillation scale for the associated wavefunctions. On the other hand, there are (associated to each of these bounded modes) a continuum, starting just above, but the delocalization should kill the overlaps with localized profiles. Of course this should be computed properly to be more quantitative.

3 Flavour violating processes

Thanks to the separation of variables, the whole set of modal wavefunctions can be decomposed as a product of a radial part⁵ and an angular one. For the fermion zero modes, the radial part is localized around the vortex⁶, while for the bosonic modes these are oscillating functions spread in the bulk. In the compactification procedure (which reduces the complete 6D theory to an effective 4D one where all modes interact among themselves), the integration over the radial component controls the strength of the interaction through the overlaps of wavefunctions, while the one over angular component gives a selection rule which forbids interactions with non zero total winding (this can be interpreted as the angular momentum conservation in the ED).

If we neglect mixing between fermions, each family is associated with one and only one winding number i . Then the interaction

$$\kappa \bar{\psi}_i \gamma^\mu \omega_{\mu,m} \psi_{i'}$$

is allowed if and only if $m = i - i'$. The strength κ depends on the radial integral⁷. Allowed effective four-fermion interactions,

$$\frac{\kappa \kappa'}{M_\omega^2} (\bar{\psi}_i O \psi_{i'}) (\bar{\psi}_j O' \psi_{j'}),$$

⁵On the sphere the angle θ plays the role of the radial variable.

⁶Note nevertheless that the size of these functions must be larger than the size of the vortex in general if we want to produce a sufficiently strong hierarchy between families (see [14] for instance).

⁷In principle, κ could be infinitely reduced by localizing more and more the fermion wavefunctions (through stronger and stronger interactions with the vortex). However as mentioned above, we are technically limited because we require (high) hierarchies between generations. We could still hope to squeeze both fermion and H fields in such a way that the hierarchy is safe, but a detailed analysis (too technical to be put in here) of the scalar sector (in the spherical case only, up to now) showed that, once m_H is fixed, we don't have this freedom anymore. Nevertheless it still is worth looking for smaller κ than imposed by the model, because we do not know what happens in a different geometry.

correspond to $(i' - i) = (j - j')$, or in other words $\Delta G = 0$, if G is some kind of family number. Thus, in first approximation (no mixing), only $\Delta G = 0$ interactions can be observed.

3.1 Forbidden kaon decays

The best experimental restriction on flavour violating processes with $\Delta G = 0$ comes from the decay $K_L^0 \rightarrow \mu^+ e^-$. In SM this process is suppressed because it is forbidden at the tree level. In our model however, there is an excited gauge mode which can mediate this decay.

To be more precise, let us focus on the spherical compactification for which we have a concrete realisation [14]. There, we have presented a set of couplings which reproduce well the SM masses and mixings as well as satisfy all constraints for masses and mixings in the neutrino sector, giving some predictions for future experiments. This realisation of the model has a fixed $R = 100$ TeV. Having all couplings fixed, we can perform quantitative calculations of all particular processes.

For any neutral gauge field W_A which interacts with the fermions, we get the following effective Lagrangian at 4D level (the scalar modes don't interact with SM fermions):

$$\mathcal{L}_{4D} \supset \sum_{\ell} \sum_{\substack{m,n \\ |n-m| \leq \ell}} E_{mn}^{\ell, |n-m|} U_{mj}^* U_{nk} (\bar{\psi}_j \gamma^{\mu} Q \psi_k) \omega_{\mu; \ell, |n-m|}^{(*)} \quad (2)$$

where $E_{mn}^{\ell, |n-m|}$ are the results of the overlaps (see [10] for details). For $\ell = 0$, we have $E_{nn}^{0,0} = 1$ (normalization) which permits to identify Q with SM charges. U is the unitary mixing matrix⁸. If it disappears properly for $\ell = 0$, this is no more the case for higher ℓ 's. Thus, in our model, it makes sense to talk about mixing in up quarks and down quarks separately, for instance. $\omega_{\mu}^{(*)}$ are the 4D fields for each modes. When $n - m \neq 0$, these are complex fields. In our notations, for $n - m > 0$ we have to use ω_{μ} , so it destroys a mode with winding $|n - m|$, while for $n - m < 0$ we have to use ω_{μ}^* , so it creates a mode with winding $|m - n|$.

K_L^0 is a combination of $\bar{s}d$ and $\bar{d}s$. The first one corresponds to indices $j = 2$ and $k = 1$ in (2). We can define matrices $\Omega_{mn}^{\ell} = U_{m2}^* U_{n1} E_{mn}^{\ell, |n-m|}$ which tell us about the strength

⁸Note that U matrices are not unique. Indeed, we could as well use $U'_L = U_L \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ and $U'_R = U_R \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ (with the same phases) since it doesn't affect the masses, but these are obviously not physical.

of coupling with each mode $\omega_{\mu;\ell,0}$, $\omega_{\mu;\ell,1}$ and $\omega_{\mu;\ell,2}$. Note that mixings in left and right sectors are different in general. For the model of [14] we have:

$$\Omega_L^\ell = \begin{pmatrix} 0.232 E_{11}^{\ell,0} & -0.057 E_{12}^{\ell,1} & 0.003 E_{13}^{\ell,2} \\ 0.941 E_{21}^{\ell,1} & -0.231 E_{22}^{\ell,0} & 0.013 E_{23}^{\ell,1} \\ -0.052 E_{31}^{\ell,2} & 0.013 E_{32}^{\ell,1} & -0.001 E_{33}^{\ell,0} \end{pmatrix} \quad \text{and} \quad \Omega_R^\ell = \begin{pmatrix} 0.053 E_{11}^{\ell,0} & -0.003 E_{21}^{\ell,1} & 0 \\ 0.997 E_{21}^{\ell,1} & -0.053 E_{22}^{\ell,0} & 0 \\ -0.001 E_{31}^{\ell,2} & 0 & 0 \end{pmatrix}$$

For both matrices, the dominant elements are (21). This means that the dominant process is the (virtual) creation of a $\omega_{\mu;\ell,1}$ (for all allowed ℓ). At first sight, it seems that the contribution to $\omega_{\mu;\ell,0}$ is significant too. But to be more precise, we have to evaluate the overlaps E^ℓ and sum over all contributions. In particular, the total contribution to $\omega_{\mu;\ell,0}$ is simply the trace (other can be obtained as sums over elements of lines parallel to the diagonal). It then is obvious (because of the unitarity of U) that this is negligible as long as $E_{11}^{\ell,0} \simeq E_{22}^{\ell,0} (\simeq E_{33}^{\ell,0})$. This result is exact for $\ell = 0$ by definition and is expected to be a good approximation for the first ℓ 's which correspond to slowly oscillating modes (thus embracing all fermion wavefunctions in a very similar way). As an example, we compute the contributions of the first modes in Table 1 (for left-handed quarks only).

ℓ	0	1	2	3	4	5
$E_{11}^{\ell,0}$	1	1.004	0.492	0.149	0.014	-0.020
$E_{22}^{\ell,0}$	1	1.073	0.496	0.027	-0.172	-0.206
$E_{33}^{\ell,0}$	1	1.419	1.268	0.923	0.603	0.374
$\omega_{\mu;\ell,0}$	0	-0.016	-0.017	0.027	0.042	0.043
$E_{12}^{\ell,1}$	/	0.780	0.872	0.621	0.359	0.186
$E_{23}^{\ell,1}$	/	0.638	0.908	0.844	0.640	0.440
$\omega_{\mu;\ell,1}^*$	/	0.742	0.832	0.595	0.346	0.181
$E_{13}^{\ell,2}$	/	/	0.051	0.027	0.018	0.013

Table 1: Overlaps between fermion pairs and first gauge modes for left down quarks. The rows $\omega_{\mu;\ell,0}$ and $\omega_{\mu;\ell,1}^*$ refer to the couplings (mixings taken into account) with these particular modes.

We can perform the same procedure for the charged lepton sector, and our previous conclusions stay more or less valid. In particular, the fact that $\omega_{\mu;\ell,0}$ don't couple much with $\bar{e}\mu$ is expected to be robust, since it depends mainly on the relative equality of all

the $E_{nn}^{\ell,0}$ for low ℓ .

We now provide the results of exact numerical evaluation at the tree level for $\Gamma(K_L^0 \rightarrow \mu^+ e^-)$ with and without mixings taken into account. Recall about the chiral suppression of this decay (angular momentum conservation imposes cancellation of the amplitude for massless fermions). Thus our result will be of the form $\Gamma \sim \beta m_\mu^2 m_K R^4 f_K^2$, where β is some dimensionless factor that accounts for the effective coupling constant which is of order $\sim (g\kappa)^4$. For a SM coupling $g \sim 10^{-1}$ and an overlap $\kappa \sim 10^{-1} \div 1$ (see Table 1), we expect $\Gamma \sim R^4 10^{-10}$. This gives a bound on R , but we remind that R plays already a role in the size of the wavefunctions, so this is only a test *a posteriori* of the validity of our choice for this parameter⁹. We could be skeptical about this rough estimation for Γ because we have to sum over all heavy modes (all ℓ 's), but remember that (in addition to overlaps reduction) we have a mass suppression $1/(\ell+1)^4 \ell^4 \sim \ell^{-8}$ which makes the series rapidly converging. Indeed, with mixing we have $\Gamma \cdot 10^{10}/R^4 = 2.24, 3.78, 4.12, 4.18$ and 4.18 for $\ell_{\max} = 1, 2, 3, 5$ and 10 respectively. Without mixing we get, for the same ℓ_{\max} , $\Gamma \cdot 10^{10}/R^4 = 3.31, 5.34, 5.72, 5.78$ and 5.78 . It gives the following limits on R :

$$\frac{1}{R} > 51(55) \text{ TeV} \quad (3)$$

with and without mixing (for the experimental limit [27] on the branching ratio $\text{Br} < 4.7 \times 10^{-22}$), which is well below the value $R = 100 \text{ TeV}$ assumed in this realisation of the model. The model with parameters of Ref. [14] (the mass of the new family-changing vector boson there is $\sim 142 \text{ GeV}$) is therefore self-consistent. To obtain a precise lower bound on R for all models, one needs to perform additional numerical work which is beyond the scope of the present note. Other rare processes may also be analyzed [28].

3.2 Collider processes

Let us briefly comment on the collider phenomenology. At the LHC, our massive bosons $\omega_{\mu;11}$ could mediate flavour violating processes if their scale is within the energy reach of the accelerator – which would assume an hypothetical geometry where $\kappa \approx 0.1$. The typical signature would be a lepton-antilepton pair with large and opposite transverse momenta. This is very similar to Drell-Yan pair production for which a typical feature is the suppression of the cross section with increasing of the resonance mass at a fixed center-

⁹Nevertheless, if we consider free κ 's, we can replace R by κR in (3).

of-mass energy. Note also that, since we are dealing here with proton-proton collisions, we expect a dominance of $(e^-\mu^+)$ and $(\mu^-\tau^+)$ over $(e^+\mu^-)$ and $(\mu^+\tau^-)$. Indeed the former processes can use valence quarks (u and d) in the proton, while the latter involve only partons from the sea.

A detailed evaluation of the expected number of events at LHC requires numerical simulation to which we will return in a future note. At this point however, it already is possible to compute the width of the $\omega_{\mu;11}$ boson thanks to (2). Note that for these energies, it is more coherent to use b_μ and ω_μ^3 instead of z_μ and a_μ . If we neglect possible model-dependent scalar interactions, we have¹⁰

$$\Gamma(b_{\mu;11} \rightarrow \text{all}) = \frac{\sqrt{2}}{R} \frac{g'^2}{32\pi} (y_e^2 A_e + 2y_L^2 A_L + y_u^2 A_u + y_d^2 A_d + 2y_Q^2 A_Q)$$

and

$$\Gamma(\omega_{\mu;11}^3 \rightarrow \text{all}) = \frac{\sqrt{2}}{R} \frac{g^2}{64\pi} (A_L + A_Q)$$

for $A = ((E_{12}^{1,1})^2 + (E_{23}^{1,1})^2)$. According to [11], we expect $\Gamma/\text{GeV} \sim \kappa^2 M/M_Z$, thus $\Gamma \sim 10^{-1} \text{ TeV}$. The exact numerical values for our example are $\Gamma(b_\mu) = 0.44 \text{ TeV}$ and $\Gamma(\omega_\mu^3) = 0.67 \text{ TeV}$.

4 Conclusions and perspectives

We have discussed the gauge sector of a successful extra-dimensional model for masses and mixing of quarks, charged leptons and neutrinos. It is important for quantitative experimental predictions of the model. Further details of the warped-geometry case will be discussed elsewhere.

We dedicate this paper to the birthday of Valery Rubakov who is not only an appreciated pioneer of large extra dimensions. He was a supervisor for two of us (M.L. and S.T.), but he is more than a teacher. He continuously sets a very high level in his studies and in the works of his school, but also in his everyday and social life. We are trying to use this level as a benchmark. Last but not least, it was Valery who initiated the first contact between J.-M.F., M.L. and S.T. in 1999, which resulted in the development of the branch discussed here.

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¹⁰We also neglect masses of all fermions and therefore mixings are irrelevant.

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